

Conic Sections CS1: Midpoint and Distance

What is the midpoint of the line segment between the two points:

1. (1, 2) & (9, 10)

$$\left(\frac{1+9}{2}, \frac{2+10}{2}\right) = \left(\frac{10}{2}, \frac{12}{2}\right) = (5, 6)$$

2. (3, 1) & (-3, 7)

$$\left(\frac{3+(-3)}{2}, \frac{1+7}{2}\right) = \left(\frac{0}{2}, \frac{8}{2}\right) = (0, 4)$$

3. (2, 2) & (6, -4)

$$\left(\frac{2+6}{2}, \frac{2+(-4)}{2}\right) = \left(\frac{8}{2}, \frac{-2}{2}\right) = (4, -1)$$

4. (1, 6) & (2, -1)

$$\left(\frac{1+2}{2}, \frac{6+(-1)}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$$

Find the distance between the two points:

5. (1, 2) & (9, 10)

$$\begin{aligned} d &= \sqrt{(9-1)^2 + (10-2)^2} \\ &= \sqrt{8^2 + 8^2} = \sqrt{64+64} \\ &= \sqrt{128} = 8\sqrt{2} \end{aligned}$$

6. (3, 1) & (-3, 7)

$$\begin{aligned} d &= \sqrt{(-3-3)^2 + (7-1)^2} \\ &= \sqrt{(-6)^2 + 6^2} = \sqrt{36+36} \\ &= \sqrt{72} = 6\sqrt{2} \end{aligned}$$

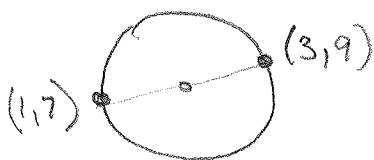
7. (2, 2) & (6, -4)

$$\begin{aligned} d &= \sqrt{(6-2)^2 + (-4-2)^2} \\ &= \sqrt{(4)^2 + (-6)^2} = \sqrt{16+36} \\ &= \sqrt{52} = 2\sqrt{13} \end{aligned}$$

8. (1, 6) & (2, -1)

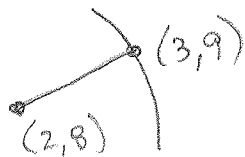
$$\begin{aligned} d &= \sqrt{(2-1)^2 + (-1-6)^2} \\ &= \sqrt{1^2 + (-7)^2} = \sqrt{1+49} \\ &= \sqrt{50} = 5\sqrt{2} \end{aligned}$$

9. A circle has a diameter with endpoints (3, 9) and (1, 7). What is the center and radius of the circle?



$$\text{Center} = \text{midpoint} = \left(\frac{1+3}{2}, \frac{7+9}{2}\right) = \left(\frac{4}{2}, \frac{16}{2}\right) = (2, 8)$$

$$\text{radius} = \text{distance} = \sqrt{(3-2)^2 + (9-8)^2}$$



$$= \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$